to its hardening under the influence of shock compression and the potential for the reversible growth of discontinuities during dynamic tension.

We thank V. É. Zgaevskii and V. K. Golubev for their discussion of the findings.

LITERATURE CITED

1. M. P. Bukhina, Theoretical Physics of Elastomers [in Russian], Khimiya, Moscow (1984).
2. A. A. Askadskii, Deformation of Polymers [in Russian], Khimiya, Moscow (1973).
3. G. I. Kapel', A. M. Molodets, and A. A. Vorob'ev, "Projection of plates by an explosion," Fiz. Goreniya Vzryva, No. 6 (1974).
4. G. I. Kanel', "Resistance of metals to cleavage fracture," Fiz. Goreniya Vzryva, No. 3 (1982).
5. J. R. Asay and L. M. Barker, "Interferometric measurement of shock-induced internal particle velocity and spatial variations of particle velocity," J. Appl. Phys., 45, No. 6 (1974).
6. M. I. Belovolov, V. I. Vovchenko, et al., "Use of laser-interferometric velocity meters in explosion experiments," Zh. Tekh. Fiz., 57, No. 5 (1987).
7. A. N. Dremin and G. I. Kanel', "Refraction of the front of an oblique shock wave at a boundary with a less stiff medium," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1970).
8. C. W. Weaver and M. S. Paterson, "Stress–strain properties of rubber at pressures above the glass transition temperature," J. Polym. Sci., 7, Pt. A-2, No. 3 (1969).
9. D. C. Erlich, D. C. Wooten, and R. C. Crewdson, "Dynamic tensile failure of glycerol," J. Appl. Phys., 42, No. 13 (1971).
10. G. W. Stepanov, Behavior of Structural Materials in Elastoplastic Loading Waves [in Russian], Naukova Dumka, Kiev (1978).
11. J. M. Walsh and M. H. Rice, "Dynamics of liquids from measurements on strong shock waves," J. Chem. Phys., 26, No. 4 (1957).
12. I. J. Ahrens and M. H. Ruderman, "Immersed-foil method for measuring shock wave profiles in solids," J. Appl. Phys., 37, No. 13 (1966).
13. F. R. Eirich and T. L. Smith, "Molecular-mechanical aspects of the isothermal rupture of elastomers," in: Fracture [Russian translation], Vol. 7, Part 2, Mir, Moscow (1976).
14. A. N. Gent and P. B. Lindley, "Internal rupture of bonded rubber cylinders in tension," Proc. R. Soc. London, 249, Ser. A, No. 1 (1959).

MOVEMENT OF THE FREE BOUNDARY OF A HALF-SPACE DURING THE PROPAGATION OF AN OBLIQUE STRAIGHT CRACK

V. A. Saraikin

UDC 539.375

Internal defects which grow dynamically in a material generate disturbances. The elastic model of the propagation of a dislocational discontinuity [1-4] is widely used in geophysics to identify the type, orientation, and size of large-scale defects – earthquake foci [1-4]. In accordance with this model, a jump in the displacement vector is assigned at the site of the discontinuity to describe the advance of the edges of the latter. This description is independent of the details of the distribution of the initial internal stress field. The orientation of the nodal planes found by this approach agrees poorly with experimental findings when the discontinuity is of the shear type. When an appropriate choice is made for the jumps in the displacements, the asymptote of the solution in the long-range field for a dislocational discontinuity differs little from the solution of the problem for a point source given by force couples without moments.

A method of describing a discontinuity (crack) which is exact within the framework of linear fracture mechanics involves specifying a drop in stress on the discontinuity [2]. The displacement field and the orientation and size of the crack in this model are consistent with the stress field, which itself conforms to the condition of dynamic instability at the crack tip.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 130-136, January-February, 1990. Original article submitted August 24, 1988.

 0021-8944/90/3101-0120$12.50

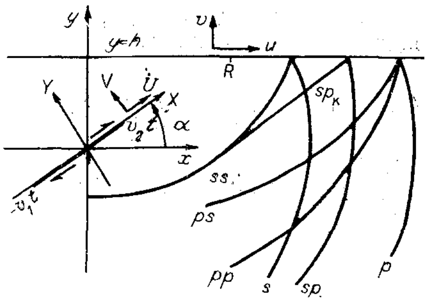

Fig. 1

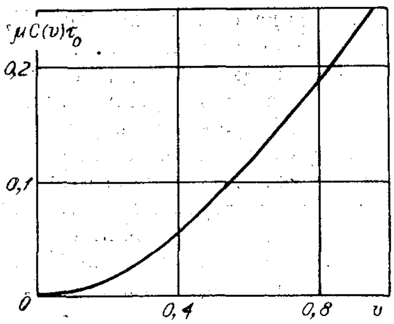

Fig. 2

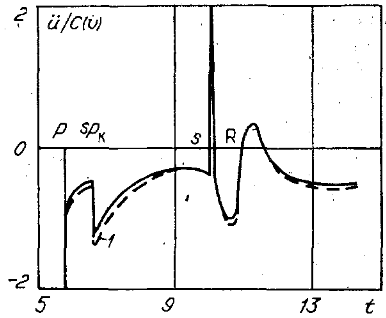

Fig. 3

Here, we examine a two-dimensional problem in a formulation which takes advantage of some of the features of the crack model mentioned above.

We will examine a half-space $y < h$ (Fig. 1) with a boundary which is free of loads. We will assume that a crack is initiated at a point located the distance h from the boundary at the moment of time $t = 9$. The crack then propagates in a straight line at the angle α to the boundary: $-v_1 t < X < v_2 t$, $Y = 0$ (X and Y are rectangular coordinates connected with the crack). It is assumed that the velocities of the tips of the crack are different but constant and less than the velocity of the Rayleigh wave ($v_i < c_R$).

The appearance of the crack inside the half-space is connected with the existence of stresses in the latter. The nature of the stresses may vary — they may be due not only to the weight of the material but also to internal disturbances or compression at infinity parallel to the boundary of the half-space. We will assume that shear stresses initially exist on the line of crack growth. These stresses can be approximated by the relation

$$\sigma_{xy}(X, 0) = \tau_n{}^0 X^n \ (n = 0, 1, 2, \ldots, \tau_n{}^0 = \mathrm{const})$$

or by a linear combination of terms of this form.

Formation of the crack leads to partial or complete dynamic relief of the shear stresses at the edges. We first find the displacements in the wave generated by the crack. Before waves arrive at the boundary, the half-space can be considered an unbounded medium. Due to the symmetry of the shear stresses relieved by the crack, the problem reduces to the solution of an auxiliary problem concerning a half-space with conditions which ensure that the sought solution has the property of similarity. These conditions are

$$\begin{aligned} &\sigma_{xy}(t, X, 0) = -\tau_n X^n \quad (-v_1 t < X < v_2 t),\\ &U(t, X, 0) = 0 \quad (X \leqslant -v_1 t,\ X \geqslant v_2 t),\\ &\sigma_{yy}(t, X, 0) = 0 \quad (-\infty < X < \infty). \end{aligned} \tag{1}$$

Here, U is a component of the displacement vector $\mathbf{U} = (U, V)$ in the coordinate system X, Y. Triviality of the normal stress σ_y means that the initial compression in the weighable half-space perpendicular to the crack growth line is assumed to be unchanging. Zero initial conditions are adopted.

Of course, this formulation does have shortcomings: 1) the motion of the crack tips is assigned rather than being determined from a fracture criterion; 2) a similarity formulation restricts the number of ways the stress relief can be specified and does not permit allowance for the contribution of body waves to the solution. This contribution is important if the growth of an initial defect of finite length is being examined. These restrictions are justified partly by the fact that the solution can be subjected to fairly thorough analysis.

In the solution of problem (1) in [5, 6], the displacement of the edges of the crack are found from the formulas

$$U(t, X,\ 0) = U_0(t, X) = \lim_{\eta\to 0}\int_0^t \frac{(t-\tau)^{n+1}}{(n+1)!}\left[W\left(\frac{\tau}{z}\right) - W\left(\frac{\tau}{\bar z}\right)\right]\frac{d\tau}{\tau},$$

$$W\left(\frac{t}{z}\right) = \frac{[(1+v_1 t/z)(1-v_2 t/z)]^{-1/2}}{2\pi i}\left\{\frac{z}{t}\int_{-v_1}^{v_2}\frac{S(\xi)\sqrt{v_1+\xi}\sqrt{v_2-\xi}}{\xi - z/t}\, f(\xi)\, d\xi + \right.$$

$$+\quad i\sum_{j=0}^{n}(-t/z)^{j}\left[\frac{A_j}{(1+v_1t/z)^{j+1}}+\frac{B_j}{(1-v_2t/z)^{j+1}}\right]\Bigg\},\quad z=X+i\eta,$$

$$f(\xi)=-\tau_n tX^n\frac{\partial^{n+1}}{\partial t^{n+1}}\left[H(v_1t+X)H(v_2t-X)\right]|_{X=\xi t},\tag{2}$$

$$S(\xi)=-\frac{ia_2^2\xi^2\sqrt{1-a_2^2\xi^2}}{\mu\left[(2-a_2^2\xi^2)^2-4\sqrt{1-a_1^2\xi^2}\sqrt{1-a_2^2\xi^2}\right]},$$

where a_1 and a_2 are the inverses of the velocities of the rarefaction and shear waves; H(...) is the Heaviside function.

Equations (1) make it possible to reduce the problem of determining the waves generated by the crack to the solution of an unmixed problem. In fact, on the crack line (Y = 0), the jump in the displacements and the stresses are equal $[U] = 2U_0(t, X)$, $[V] = [\sigma_{yy}] = [\sigma_{xy}] = 0$. It follows from this that the displacement U is antisymmetric, while V is symmetric relative to the crack line. Thus, the displacements outside the crack can be found by combining the solutions of the two problems for the upper (Y > 0) and lower half-spaces with the boundary conditions

$$U(t, X, \pm 0) = \pm U_0(t, X),\ \sigma_{yy}(t, X, 0) = 0.$$

Using a Laplace transform in t and a Fourier transform along X, we find the LF-transforms of the displacements in the wave coming from the crack

$$U^{LF}(s, sp, Y)=-\frac{w(p)}{a_2^2s^{n+3}}\left[2p^2e^{-sn_1|Y|}-(n_2^2+p^2)e^{-sn_2|Y|}\right]\operatorname{sgn}(Y),$$

$$V^{LF}(s, sp, Y)=\frac{ipw(p)}{a_2^2s^{n+3}}\left[2n_1n_2e^{-sn_1|Y|}-(n_2^2+p^2)e^{-sn_2|Y|}\right],\tag{3}$$

$$n_i^2(p)=a_i^2+p^2.$$

Here, s and sp are parameters of the Laplace and Fourier transforms, respectively; $s^{-n-3}w(p)$ is the transform of the displacement of an edge of the crack $U_0(t, X)$.

The next step is to account for the interaction of the wave (3) with the boundary of the half-space. This can be done more simply if we make the substitution of coordinates $x + iy = (X + iY)e^{i\alpha}$ (see Fig. 1) and change over in (3) from the Fourier transform F in the variable X to the transformation $\mathcal{F}$ of the same solution in the variable x.

Having used sq to represent the parameter of the Fourier transform in x, after recalculating we find that Eqs. (3) take the form

$$U^{L\mathcal{F}}(s, sq, y)=\frac{1}{\pi ia_2^2s^{n+3}}\int_{-\infty}^{\infty}\left(\frac{2p^2}{N_1}-\frac{a_2^2+2p^2}{N_2}\right)(p\cos\alpha-q)\,w(p)\,e^{-isMy}dp,$$

$$V^{L\mathcal{F}}(s, sq, y)=-\frac{\sin\alpha}{\pi ia_2^2s^{n+3}}\int_{-\infty}^{\infty}\left(2\frac{a_1^2+p^2}{N_1}-\frac{a_2^2+2p^2}{N_2}\right)pw(p)\,e^{-isMy}dp,$$

$$M=(p-q\cos\alpha)/\sin\alpha,\ N_i=n_i^2(q)\sin^2\alpha+(p-q\cos\alpha)^2,$$

$$\begin{Bmatrix}U\\V\end{Bmatrix}(t, x, y)\equiv\begin{Bmatrix}U\\V\end{Bmatrix}(t, \operatorname{Re}(x+iy)e^{-i\alpha}, \operatorname{Im}(x+iy)e^{-i\alpha}),$$

$$u+iv=(U+iV)e^{i\alpha}.$$

The last row shows the formula for the transition from the displacements U, V in the coordinate system X, Y to the displacements u, v along the x, y axes.

Now knowing $L\mathcal{F}$ – the transforms of the displacements in a wave incident on the free boundary of the half-space, we solve the dynamic problem for the half-space y < h to find the transforms of the displacements in the reflected wave. The solution is represented in the form of integrals calculated from residues. The surface integrals obtained in the calculations can be discarded, since they are equal to zero for the moments of time prior to the arrival of the crack tip at the boundary of the half-space. Omitting these cumbersome calculations, we come to the final result for the displacements on the boundary of the half-space [7, 8]:

$$a_2^2 s^{n+3} u^{L\mathcal{F}}(s, sq, h) = \frac{iQ_1}{n_1}\left(\underline{1} - 1 + \frac{4a_2^2 n_1 n_2}{R}\right) e^{-sn_1 h} + Q_2\left(\underline{1} - 1 + \frac{2a_2^2\left(a_2^2 + 2q^2\right)}{R}\right) e^{-sn_2 h},$$

$$a_2^2 s^{n+3} v^{L\mathcal{F}}(s, sq, h) = Q_1\left(\underline{1} - 1 + \frac{2a_2^2\left(a_2^2 + 2q^2\right)}{R}\right) e^{-sn_1 h} - \frac{iq}{n_2} Q_2\left(\underline{1} - 1 + \frac{4a_2^2 n_1 n_2}{R}\right) e^{-sn_2 h},$$

$$Q_1(q) = [(n_1^2 + q^2)\sin 2\alpha + 2iqn_1 \cos 2\alpha] w(q\cos\alpha - in_1 \sin\alpha),$$

$$Q_2(q) = [(n_2^2 + q^2)\cos 2\alpha - 2iqn_2 \sin 2\alpha] w(q\cos\alpha - in_2 \sin\alpha), \quad (4)$$

$$R = (n_2^2 + q^2)^2 - 4q^2 n_1 n_2, \ n_i^2 = a_i^2 + q^2.$$

We have underlined the terms corresponding to the transforms of the displacements in the wave traveling toward the boundary.

At points of the boundary, solution (4) is valid until the arrival of secondary waves – waves generated by the crack and waves reflected from the boundary and then the crack, thus returning to the boundary. In order to account for the contribution of the secondary waves to the solution, it is necessary to solve a nonsimilar problem concerning diffraction of waves at the crack. It should be noted that due to the reduction in the amplitude of a wave as it travels in the medium, the contribution of secondary waves to the solution at the boundary will evidently be less important than the contribution of the primary waves.

Equations (4) account for their uniformity relative to the parameters of the Laplace and Fourier transforms. The originals for transforms of these types can be obtained in explicit form [6, 9]

$$u(t, x, h) = \frac{1}{\pi}\operatorname{Re}\int_0^t \left\{\frac{4i\xi_1 n_2(\xi_1) Q_1(\xi_1)}{R(\xi_1)}\frac{\partial \xi_1}{\partial \tau} + \frac{2\left(a_2^2 + 2\xi_2^2\right) Q_2(\xi_2)}{R(\xi_2)}\frac{\partial \xi_2}{\partial \tau}\right\} \times \frac{(t-\tau)^{n+1}}{(n+1)!}\, d\tau,$$

$$\xi_i = (x^2 + h^2)^{-1}\begin{cases} -i\tau x + h\sqrt{r_i} & (r_i \geqslant 0), \\ -i\tau x + ih\sqrt{-r_i}\ \operatorname{sgn}(x) & (r_i < 0,) \end{cases}$$

$$n_i(\xi_i) = (x^2 + h^2)^{-1}\begin{cases} \tau h - ix\sqrt{r_i} & (r_i \geqslant 0), \\ \tau h + |x|\sqrt{-r_i} & (r_i < 0), \end{cases} \quad (5)$$

$$r_i = \tau^2 - a_i^2(x^2 + h^2).$$

The values of the real and imaginary parts of the remaining radicals satisfy the inequalities $\operatorname{Re} n_2(\xi_1) \geqslant 0,\ x \operatorname{Im} n_2(\xi_1) \leqslant 0,\quad \operatorname{Re} n_1(\xi_2) \geqslant 0,\ x \operatorname{Im} n_1(\xi_2) \leqslant 0$. The displacement v is calculated from a similar formula.

We will present some results of calculations of solution (5) for the case when the shear stresses relieved by the propagation of the crack are constant ($\tau_0 = \tau_0^0$ = const, n = 0) and the crack tips move in different directions at the same velocity ($v_1 = v_2 = v$). In this case, in Eq. (4) we should put

$$w(p) = \frac{2\pi a_2^2 C(v)}{(1 + v^2 p^2)^{3/2}}, \quad C(v) = \frac{\tau_0 v^4 q_2^2}{D(v)},$$

$$D(v) = (4a_2^2 v^2 q_2^2 + a_2^4 v^4) K(q_2) - 4a_1^2 v^2 q_2^2 K(q_1) + 8q_2^2 E(q_1) - \quad (6)$$

$$(8q_2^2 + a_2^4 v^4) E(q_2), \ q_i^2 = 1 - a_i^2 v^2.$$

A graph of the function C(v) is shown in Fig. 2. Here and below, we took the following as the unit of measurement: the distance h from the center of the crack to the boundary; the velocity of the shear wave $c_2 = a_2^{-1} = 1$. The Poisson's ratio was equal to 0.25, so that $c_1 = a_1^{-1} = \sqrt{3}$.

The solid lines in Figs. 3 and 4 show the change in accelerations over time on the surface of the half-space at the point x = 10, y = 1 with an angle of inclination of the crack $\alpha = \pi/4$. The curves were constructed for a crack propagating at a velocity v = 0.25. The symbols p and s denote the moments of arrival of the fronts of the rarefaction and shear waves. The combination of symbols denotes the position of the fronts of the reflected waves (see Fig. 1). The first symbol indicates the type of incident wave, while the second indicates the type of wave reflected from the boundary. The symbol R denotes the position of the front of the Rayleigh wave, while sp_1 denotes a lateral wave.

The solid line in Fig. 5 shows the displacement of this point. Dimensionless values of the displacements, referred to the constant C(v) = C(0.25), are plotted off the axes. Time was chosen as the parameter which changes along the curve. The points s and R on the curves denote the moments of arrival of the shear and Rayleigh waves.

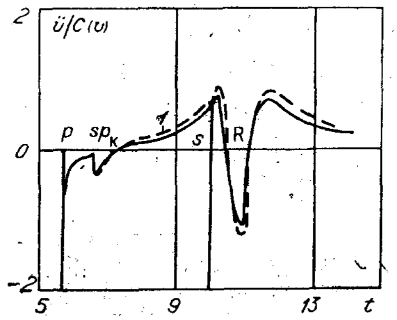

Fig. 4

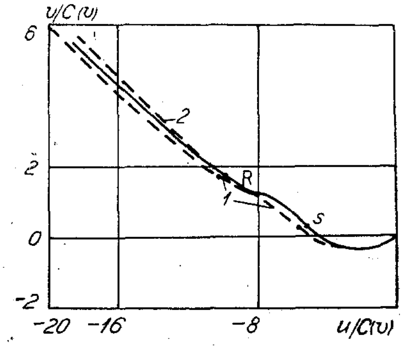

Fig. 5

Let us evaluate the dimensional displacements, which can be obtained by calculation. In Fig. 2, for $v = 0.25$ we have $C(v) \approx 0.02\tau_0/\mu$. We approximately calculate the scale of the grid in Fig. 5, assuming that the origin is located at the depth $h = 10^3$ m. The specific weight of the medium $\gamma = 1.5 \cdot 10^{-4}$ N/m^3. Evaluating the shear stresses relieved by the crack, we find

$$\sigma_{xx}^0 \sim \frac{\nu}{1-\nu}\sigma_{yy}^0 \sim -\nu\gamma\frac{h}{1-\nu} = -5 \text{ MPa}, \quad |\tau_0| \sim |\sigma_{yy}^0 - \sigma_{xx}^0|/2 \sim 5 \text{ MPa}.$$

It is assumed that the shear stress is completely removed. Taking the shear modulus μ to be equal to 10^4 MPa, we find that the unit of the grid scale corresponds to the displacement $u = C(v)h \sim 10^{-2}$ m. The units of the scales in Figs. 3 and 4 correspond to time and acceleration ($c_2 \sim 3000$ m/sec): $t = h/c_2 \sim 1/3$ sec, $\ddot{u} = C(v)c_2^2/h \sim 0.1$ m/sec^2.

Let us compare these results with the results found from the dislocational model of rupture. The changes in solution (4) concern the source functions. In the example being examined, the displacement of the edges of the crack

$$U_0^{LF}(s, sp) = \frac{w(p)}{s^3}, \quad U_0(t, X) = 2C(v)\, a_2^2 v^{-2}\sqrt{v^2t^2 - X^2}. \tag{7}$$

As an equivalent dislocation source, we take a discontinuity with a displacement jump which is constant along its length $U = U_d H(v_d t - |X|)$ and a size of jump $U_d(t)$ determined from the condition of equality of the "volumes":

$$\int_{-vt}^{vt} U_0(t, X)\, dX = \int_{-v_d t}^{v_d t} U_d(t)\, dX, \tag{8}$$

from which $U_d(t) = \pi a_2^2 C(v) t^2/2v_d$ and, thus,

$$U^{LF}(s, sp) = \frac{w_d(p)}{s^3}, \quad w_d(p) = \frac{2\pi a_2^2 C(v)}{(1 + v_d^2 p^2)^2}\frac{v}{v_d}.$$

Comparing these expressions with (6) and (7), we see that in solution (4) we need to replace w by w_d.

Calculations performed for $v_d = 0.9$ show that the accelerations and displacements differ little from the curves obtained for source (7) (the dashed curves 1 in Figs. 3-5). If the velocities of the discontinuity v_d and the crack v are the same, then the situation is such as to ensure constant satisfaction not only of Eq. (8), but also equality of the mean shifts

$$U_{\text{av}}(t) = \frac{1}{2vt}\int_{-vt}^{vt} U_0(t, X)\, dX = \frac{U_d(t)}{2vt}.$$

Calculations show that the accelerations in this case are nearly the same as the values found for the crack. The dashed line in Fig. 5 shows the trajectory of the displacement of a point on the free surface. The difference between the solutions is negligible. This may be connected with the similar description of the two solutions. Here, the dynamics of the exchange of waves between tips was established, and the high-frequency part of the spectrum is not as representative as in unsteady problems.

LITERATURE CITED

1. A. V. Vvedenskaya, Investigation of Stresses and Ruptures at Earthquake Foci by Means of the Theory of Dislocations [in Russian], Nauka, Moscow (1969).
2. B. V. Kostrov, Mechanics of the Foci of Tectonic Earthquakes [in Russian], Nauka, Moscow (1978).
3. J. Rice, Mechanics of Earthquake Foci [Russian translation], Mir, Moscow (1982).
4. B. V. Kostrov and L. V. Nikitin, "Generation of elastic waves in the disruption of the continuity of an elastic medium. Plane problem," Izv. Akad. Nauk SSSR, Fiz. Zemli, No. 7 (1968).
5. B. V. Kostrov, "Similarity problems on the propagation of a shear crack," Prikl. Mat. Mekh., 28, No. 5 (1964).
6. L. I. Slepyan, Fracture Mechanics [in Russian], Sudostroenie, Leningrad (1981).
7. V. A. Saraikin, "Motion of the boundary of a half-plane caused by a growing crack," Dokl. Akad. Nauk SSSR, 273, No. 3 (1983).
8. V. A. Saraikin, "Displacement of the boundary of a half-space in the propagation of a longitudinal shear crack at an angle to the boundary," in: Summary of Documents of a Conference on the Propagation of Elastic and Elastoplastic Waves, Part 2, Frunze (1983).
9. L. I. Slepyan, Unsteady Elastic Waves [in Russian], Sudostroenie, Leningrad (1972).

BRANCHING METHODS OF ANALYZING A DISTURBANCE OF THE CRITICAL-PRESSURE SPECTRUM OF SHELLS OF REVOLUTION AND SOME APPLICATIONS OF THESE METHODS

V. V. Larchenko

UDC 539.3:534.1

New phenomena in the branching loss of stability of an elastic shell were discovered in a study conducted for a singular perturbation. It was found in particular that disturbance of the middle surface and the load is accompanied by a change in the type of bifurcation of the branch points, the rotation group of the minor equilibrium mode, and the multiplicity of the eigenvalues. Conditions were formulated for the functionals of the branching equation for which the multiplicity increases to the specified value. This makes it possible to significantly simplify the theory of models of instability. To establish the above facts, it is important that the spectrum be crowded at $\mu \to 0$ (μ is a natural small parameter with higher derivatives). A theoretical-empirical method of evaluating the effectiveness of electrophysical loading of thin shells was proposed within the framework of the completed study.

1. Let (r, φ) be a polar coordinate system whose origin is located at the tip of a shallow spherical segment. We will examine the below Marguerre–Vlasov problem [1-3] in the space

$$\begin{gathered}\mu^{2(3-k)}\Delta^2 w = \theta\Delta\Phi + \mu^q[L(w, \Phi) + L(w_\tau, \Phi)] + \mathscr{P}(r),\ (r, \varphi) \in \Omega,\\ \mu^{2(k-1)}\Delta^2\Phi = -\theta\Delta w - \mu^q[L(w, w) + 2L(w, w_\tau)]/2,\\ w = w' = 0,\ A\Phi = B\Phi = 0,\ r \in \partial\Omega,\\ rL(u,\ v) = u''Av + v''Au - 2r^{-1}BuBv,\ A(\cdot) = (\cdot)' + r^{-1}(\cdot)^{\cdot\cdot},\\ B(\cdot) = [(\cdot)' - r^{-1}(\cdot)]^{\cdot},\ \mathscr{P}(r) = p + \delta\eta(r),\ p \in \{p_n\},\ |\delta| \ll 1.\end{gathered} \tag{1.1}$$

Here, w is the normal displacement of the middle surface; Φ is the Airy stress function; $\mathscr{P}(r)$ is the external pressure; $\{p_n\}$ is a sequence of critical pressure of a perfectly spherical dome; δ is the density of the pressure disturbance; $\mu^2 = h/a\gamma$ is a natural small parameter; h is the thickness; $2a = \operatorname{diam}\Omega$; $\gamma^2 = 12(1 - \nu^2)$; $\nu \in (0, 0, 5)$ is the Poisson's ratio; θ is a half-angle; $w_\tau(r, \varphi)$ is a 2π-periodic disturbance of the middle surface such that $w_\tau(r, \varphi) \in \mathscr{F}_\tau(\Omega)$, $|\tau_n| \ll 1$, $\|f_n(r)\|_c = 1$, where

$$\mathscr{F}_\tau(\Omega) = \left\{f_\tau \in \mathring{H}^2(\Omega)\,|\,f_\tau(r, \varphi) = \sum_{n=1}^{\infty} f_n^\tau(r)\cos n\varphi,\quad f_n^\tau(r) = \tau_n f_n(r)\right\}.$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 136-143, January-February, 1990. Original article submitted May 16, 1988; revision submitted August 24, 1988.

